Passage d’un modèle d’endommagement continu régularisé à un modèle de fissuration cohésive dans le cadre de la rupture quasi-fragile

Soutenance de thèse de Sam Cuvilliez
01/02/2012

Directeur de thèse :
Frédéric Feyel

Encadrant industriel :
Sylvie Michel-Ponnelle
Introduction

1D analytical study: transition from non local damage growth to cohesive crack opening

Extension to a 2D/3D FE framework

Conclusion and prospects
Introduction
- Industrial context / scientific context
- Continuous and discontinuous approaches for fracture

1D analytical study: transition from non local damage growth to cohesive crack opening
- Continuous reference solutions for two regularization techniques
- Identification of an equivalent cohesive law
- Switching from non local damage to cohesive zone model

Extension to a 2D/3D FE framework
- FE implementation: explicit coupling algorithm
- 2D simulations
- 3D simulations

Conclusion and prospects
Introduction
Numerical simulation of crack initiation and propagation in several industrial structures of EDF’s electricity generation fleet

- we focus on brittle / quasi-brittle material
- large (reinforced) concrete structures (civil engineering structures)
- irradiated steel components (reactor vessel…)

**Code_Aster:** an open-source FE software developed by EDF R&D

→ including fracture and damage mechanics capabilities for structural analysis
Scientific project:

- combining continuous / discontinuous fracture models
  → from damage inception to macro-crack propagation
- partnership between Centre des Matériaux, ONERA, EDF R&D (2 PhD theses)
- this work is complementary to S. Feld-Payet thesis (ONERA/ENSMP) [Feld-Payet 2010]

- ZéBuLoN (ONERA/Centre des Matériaux FE software)
- elasto-plasticity coupled with non local damage
- mesh adaptivity
- 2D/3D situations
- insertion of cracks at complete damage:
  → crack path tracking based on the gradient of damage
  → insertion of discontinuity surfaces with an explicit remeshing technique
Several ways to model fracture in FE analysis:

- Global approaches (energy balance)
  - valid for pre-existing cracks
  - inability to deal with crack onset (propagation criterion only)

- Continuum Damage Mechanics (CDM)
  - continuous description of fracture mechanisms
  - ability to model “crack” onset and propagation

- Cohesive zone models (CZM)
  - discontinuous description of fracture mechanisms
  - ability to model crack onset and propagation (limitations concerning the crack path)
A simple isotropic brittle damage model

- Rigidity loss is described through an irreversible scalar damage variable $d \in [0,1]$

\[ d = 0, \text{ sound material} \]
\[ 0 < d < 1, \text{ partial damage} \]
\[ d = 1, \text{ broken material} \]

- The elastic stress–strain relation is enhanced by a function of $d$: $A(d)$

\[ \sigma = A(d)E : \varepsilon \]

$A([0,1]) = [0,1]$  
$A(0) = 1$ and $A(1) = 0$  
$A' < 0$ ...  
usually, $A(d) = 1 - d$
**generalized standard materials framework** [Halphen & Nguyen, 1975]

- positive and convex potentials, thermodynamically admissible constitutive laws

\[
\phi(\varepsilon, d) = \frac{1}{2} A(d) \varepsilon : E : \varepsilon
\]

\[
\Psi(\dot{d}) = k \dot{d}
\]

- **σ – ε relation**

\[
\sigma = A(d) E : \varepsilon
\]

- **damage driving force**

\[
Y = -\frac{1}{2} A'(d) \varepsilon : E : \varepsilon
\]

- **threshold function**

\[
f(Y) = Y - k
\]

- **loading/unloading conditions**

\[
f \leq 0 ; \quad \dot{d} \geq 0 ; \quad \dot{d} f = 0
\]
Rate-independent constitutive laws + softening = localization

- loss of ellipticity of the equilibrium rate equations

Numerical consequences: severe mesh dependency

- strain and damage fields concentrate in a band of 1 element thickness
- the band location is dependent on the mesh orientation

Ex: trapezoidal DCB specimen

⇒ the amount of dissipated energy is directly related to the local mesh size
Rate-dependent approaches [Needleman, 1988]…

- introduction of viscosity in the constitutive equations

“non local” damage models: spatial localization limiters

- integral / differential non local models
  → equivalence [Peerlings, 1999]

- enriched kinematics / gradient of internal variable
  approaches
  → equivalence [Lorentz & Andrieux, 1999]

- higher order continuum (2\textsuperscript{nd} gradient theories)

[Forest, 2009] : this approaches are specific cases of a same unifying thermomechanical framework ("Micromorphic Approach")
### Continuum Damage Mechanics

#### non local damage models (2)

**local constitutive law**

\[
\sigma = A(d)E : \varepsilon \\
Y = -\frac{1}{2}A'(d)\varepsilon : E : \varepsilon \\
f (Y) = Y - k
\]

**differential implicit non local model** [Peerlings et al., 1996]

\[
\bar{Y} = -\frac{1}{2}A'(d)\bar{\varepsilon} : E : \bar{\varepsilon}
\]

with \(\bar{\varepsilon}\) such that

\[
\begin{align*}
\bar{\varepsilon} - \ell^2\nabla^2 \bar{\varepsilon} &= \varepsilon \quad \text{in } \Omega \\
\nabla \bar{\varepsilon} \cdot n &= 0 \quad \text{on } \partial\Omega
\end{align*}
\]

**gradient damage model** [de Borst & Mühlhaus, 1992], [Lorentz & Andrieux, 1999]

\[
\phi(\varepsilon, d, \nabla d) = \frac{1}{2}A(d)\varepsilon : E : \varepsilon + \frac{c}{2}(\nabla d)^2
\]

\[
\bar{Y} = -\frac{1}{2}A'(d)\varepsilon : E : \varepsilon + c\nabla^2 d
\]
Cohesive Zone Models (CZM)

- Halfway between Griffith’s theory and CDM:
  - damage variable is lumped on the crack surface $\Gamma$
  - stress singularity near the crack tip is removed → existence of a “cohesive process zone”
  - able to deal with crack onset without initial geometric singularities

$$E_{pot}(u, \delta) = E_{vol}(u) - W_{ext}(u) + \int_{\Gamma} \psi(\delta) \, d\Gamma$$

- Separation law $\tilde{\sigma}(\delta)$ is derived from the surface energy density $\psi(\delta)$
- 2 parameters: critical stress $\sigma_c$ and fracture energy $G_c$
Combining both continuous and discontinuous descriptions

- **transition at** $d = 1$ : insertion of traction-free discontinuities
  
  [Simone *et al.* 2003], [Mediavilla *et al.* 2006], [Feld-Payet 2010]

- **transition at** $d \leq 1$ : energy transfer to CZM
  
  [Comi *et al.* 2007], [Cazes *et al.* 2008], [Simatos *et al.* 2010], [Jirásek & Zimmermann 2001]

- 😊 crack path prediction (inherent to the model)
- 😇 damage concentration ≠ real crack
- 😊 only valid for a postulated crack path
- 😇 cohesive crack = real material separation

brittle fracture
Introduction
- Industrial context / scientific context
- Continuous and discontinuous approaches for fracture

1D analytical study: transition from non local damage growth to cohesive crack opening
- Continuous reference solutions for two regularization techniques
- Identification of an equivalent cohesive law
- Switching from non local damage to cohesive zone model

Extension to a 2D/3D FE framework
- FE implementation: explicit coupling algorithm
- 2D simulations
- 3D simulations

Conclusion and prospects
1D analytical study: transition from damage growth to cohesive crack opening
Brittle tensile bar: inhomogeneous localized solution

- monotonically increasing load
- load parameter: top damage value \( d_0 = d(0) \)
- localization at the centre of the bar
  → symmetry: the problem is solved in \([0,L]\)

**equilibrium:**
\[
\sigma'(x) = 0
\]

**stress – strain relation:**
\[
\sigma = A(d)E\varepsilon
\]

**damage evolution equations:**
\[
\begin{cases}
  f = 0 \text{ and } \dot{d} > 0 \text{ in } [0,b(d_0)] \\
  f < 0 \text{ and } \dot{d} = 0 \text{ in } [b(d_0),L]
\end{cases}
\]

→ if the bandwidth \( b \) strictly increases with \( d_0 \), irreversibility condition is automatically ensured and the problem reads:
\[
f = 0 \text{ in } [0,b(d_0)]
\]
gradient damage model: [Pham & Marigo 2010], [Lorentz & Godard 2011]

\[ c \nabla^2 d - \frac{1}{2} A'(d) \varepsilon E \varepsilon - k = 0 \]

strictly increasing bandwidth with:

\[ A(d) = \left( \frac{1-d}{1+\gamma d} \right)^2 ; \gamma > 0 \]

[Lorentz & Godard 2011]

non linear ODE to solve

corresponding damage profiles:

"2D" = 2b(1)

\( d_0 = 1 \)
**Closed-form solution**

**implicit gradient-enhanced model**

\[ \varepsilon - \ell^2 \nabla^2 \varepsilon = \varepsilon \]

constraints on the rigidity function \( A(d) \):

- calculate antiderivatives of \( S = 1/A \)
- bandwidth \( b(\varepsilon_0) \) must be strictly increasing with \( \varepsilon_0 \)

\[ A(d) = \left( \frac{1 - d}{1 + \gamma d} \right)^{p/3}, \ p \in \mathbb{N}, \ p \geq 6 \]

non linear ODE to solve

first integral: analytical expression of \( \sigma(\varepsilon_0) \)

nondimensionalization

integration by RK4 procedure
Closed-form solution
comparison between both regularization techniques

- same local brittle law:
  \[
  \begin{align*}
  \sigma &= A(d)E\varepsilon \\
  Y &= -\frac{1}{2} A'(d)\varepsilon E\varepsilon \\
  f(Y) &= Y - k
  \end{align*}
  \]

- same set of material parameters:
  \[
  \begin{align*}
  E &= 3 \times 10^4 \text{ MPa} \\
  \sigma_y &= 3 \text{ MPa} \\
  G_f &= 9.32 \times 10^{-2} \text{ N/mm} \\
  2D &= 116 \text{ mm}
  \end{align*}
  \]
  (concrete type material)

- 2 regularization techniques:
  \[
  \bar{Y} = -\frac{1}{2} A'(d)\bar{\varepsilon} E\bar{\varepsilon} \quad \rightarrow \text{implicit gradient formulation}
  \]
  \[
  \bar{Y} = -\frac{1}{2} A'(d)\varepsilon E\varepsilon + c\nabla^2d \quad \rightarrow \text{gradient damage formulation}
  \]
Closed-form solution
comparison between both regularization techniques

- in terms of load/displacement curve
  - stress decreases from $\sigma_y$ to 0
  - elasticity, critical stress and dissipated energy are the same!
Closed-form solution
comparison between both regularization techniques

- in terms of damage profiles
  - implicit gradient model: spurious widening of the fully damaged band [Geers et al., 1998]
  - gradient damage model: sharp profile → failure is reached on a single point
Closed-form solution
comparison between both regularization techniques

- spurious growth of the localisation bandwidth
  - this issue is inherent to the regularization technique (\( \bar{\varepsilon} \) profiles)
  - consequences on the load/displacement curve
Closed-form solution
comparison between both regularization techniques
Cohesive law $\delta^{EQ}(\sigma)$ is constructed by enforcing the equality:

$$U^{CDM} = U^{CZM} \quad \forall \sigma$$

with

$$\delta^{EQU}(\sigma) = \frac{2\sigma}{E} \left( b - \int_{0}^{b} \frac{dx}{A(d(x))} \right)$$
Transition to CZM is triggered at an arbitrary load level corresponding to a “critical” damage value $d_{cr}$.

$$U_{t_{cr}}^{CDM} = U_{t_{cr}}^{CZM} \quad \forall t > t_{cr}$$

**Equivalent cohesive law**

$$\delta_{SWI}^{(\sigma_t)} = \delta_{EQU}^{(\sigma_t)} - \delta_{COR}^{(\sigma_t)}$$

« switch » cohesive law  

**Corrective term**

$\delta_{cr}$  

$\sigma$ (MPa)  

$\delta$ (mm)
switching from damage growth to cohesive crack opening (2)
$2L = 2000 \text{ mm}$

**Parameters**

- $E = 30 \text{ GPa}$
- $\sigma_y = 3 \text{ MPa}$
- $G_f = 0.1 \text{ N/mm}$
- $\nu = 0$
- $2D = 100 \text{ mm}$

**1st FE test case: implementation assessment**

- **Prescribed displacement (mm)**
- **Resultant force (N/mm)**
- **Switch for $d_{cr} = 0.3$**

**Graphs**

- **Non-local**
  - Various values of $h$ vs. prescribed displacement
  - Analytical solution

- **Switch for $d_{cr} = 0.3$**
  - FEM: $h = 2D/16$
  - Analytical solution
  - Switch, $d_{cr} = 0.3$
For a given non local damage model, an **energetically equivalent** continuous / discontinuous approach can be constructed:

**1st step:**
- CDM
- CZM

**2nd step:**
- CDM
- CZM

Can this strategy be extended to 2D / 3D situations?
- no analytical solutions ⇒ FE analysis
- 1D analysis ⇒ construction of mode I cohesive law
- for a mode I straight crack propagation, does the gradient damage model locally behave like in 1D?
Extension to 2D/3D situations?

- stand-alone non local 2D FE simulation

\[ \nabla \cdot e \]

\[ \sigma_{yy} \]

\[ d \]

\[ |\nabla d \cdot e_x| \]

\[ |\nabla d \cdot e_y| \]
Introduction
- Industrial context / scientific context
- Continuous and discontinuous approaches for fracture

1D analytical study:
transition from non local damage growth to cohesive crack opening
- Continuous reference solutions for two regularization techniques
- Identification of an equivalent cohesive law
- Switching from non local damage to cohesive zone model

Extension to a 2D/3D FE framework
- FE implementation: explicit coupling algorithm
- 2D simulations
- 3D simulations

Conclusion and prospects
Extension to 2D/3D FE element framework
Extend the 1D analytical results to crack propagation

- **Assumptions:** rectilinear crack propagation under mode I loading condition
  - **Rectilinear:** *a priori* known crack path
    - The cohesive interface is inserted in the discretisation before the computation (but not activated)
  - **Mode I:** one-dimensional results can be transposed in the normal direction to the crack path to locally ensure energy transfer from volume to cohesive interface
Explicit coupling algorithm

Cohesive behaviour

- Cohesive interface initially set to adherence regime
- Identification and storage of the appropriate cohesive law for each cohesive Gauss point
- Treatment of the non local vicinity of each activated cohesive element

Local activation criterion
(at the cohesive Gauss point scale)

- Non linear time step in $t^+$
- $t^+ = t^* + \Delta t$
- $d \geq d_{cr}$?

- current cohesive Gauss point
- closest Gauss point in each neighboring non local element

ADHERENCE

$\sigma_n \rightarrow \delta_n$

ACTIVATED COHESIVE BEHAVIOUR

$\sigma_n \rightarrow \sigma_c \rightarrow G_c \rightarrow \delta_c \rightarrow \delta_n$
Discretisation

$E = 30$ GPa
$\sigma_y = 3$ MPa
$\nu = 0.2$
$G_f = 0.1$ N/mm
$2D = 100$ mm

(concrete type material)

Numerical examples
trapezoidal DCB specimen: stable crack propagation (1)
Numerical examples
trapezoidal DCB specimen: stable crack propagation (2)

animation #1

ZOOM

trapezoidal DCB specimen: stable crack propagation (2)

Numerical examples
trapezoidal DCB specimen: stable crack propagation (2)

29 – 01/02/2012
Very slight dependency on the critical parameter value $d_{cr}$ in terms of global response.

Influence of the critical parameter value $d_{cr}$:
- on the cohesive process zone (cpz) length
- on the traction-free discontinuity (tfd) length
Computational cost: non local vs. coupled approach

<table>
<thead>
<tr>
<th>number of newton iterations per time step</th>
<th>non local, $d_{cr} = 0.1$</th>
<th>coupled, $d_{cr} = 0.1$</th>
<th>coupled, $d_{cr} = 0.3$</th>
<th>coupled, $d_{cr} = 0.5$</th>
<th>coupled, $d_{cr} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>minimum</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>average</td>
<td>7.7</td>
<td>8.5</td>
<td>7.8</td>
<td>7.5</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Comparison for $h=2D/10$ (122,870 dofs)

- The coupled approach does not generate additional CPU cost whatever the critical damage parameter value
  - No additional balancing time step consecutive to discontinuity insertion
  - No additional Newton iterations per time step
Numerical examples
perforated plate specimen: unstable multi-crack propagation

animation #2
Numerical examples
Compact Tension (CT) test specimen (1)

CT specimen: geometry (mm)

Material behaviour
- steel at -150°C ⇒ brittle behaviour
- elasticity + brittle damage (plasticity is neglected)
- experimental parameters (CEA)
  \[ E = 210 \text{ GPa} \quad \sigma_c = 1.2 \text{ GPa} \]
  \[ \nu = 0.3 \quad G_c = 528 \text{ N/mm} \]
- ultimate damage bandwidth (numerical parameter)
  \[ 2D = 0.3 \text{ mm} \]
- corresponding “equivalent cohesive law”
Numerical examples
Compact Tension (CT) test specimen (2)

simulated load-CMOD curves

- non local
- CZM
- coupled $d_{cr} = 0.7$

deformed shape at crack inception

pre-crack tip position

CDM

CZM

coupled
Numerical examples
Compact Tension (CT) test specimen (2)

simulated load-CMOD curves

![Simulated Load-CMOD Curves]

experimental load-CMOD curves

![Experimental Load-CMOD Curves]

comparsion with CEA experimental results

<table>
<thead>
<tr>
<th></th>
<th>exp.</th>
<th>CDM</th>
<th>CZM</th>
<th>coupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>peak force</td>
<td>8.1 kN</td>
<td>8.13 kN</td>
<td>7.84 kN</td>
<td>8.09 kN</td>
</tr>
<tr>
<td>rel. error on</td>
<td>11.0 %</td>
<td>7.80 %</td>
<td>11.6 %</td>
<td>11.0 %</td>
</tr>
<tr>
<td>peak force</td>
<td></td>
<td>0.140 mm</td>
<td>0.136 mm</td>
<td>0.140 mm</td>
</tr>
<tr>
<td>correspond CMOD</td>
<td>0.126 mm</td>
<td>0.140 mm</td>
<td>0.136 mm</td>
<td>0.140 mm</td>
</tr>
<tr>
<td>rel. error on CMOD</td>
<td></td>
<td>11.6 %</td>
<td>7.80 %</td>
<td>11.0 %</td>
</tr>
</tbody>
</table>
Straightforward extension of the 2D implementation to 3D plane crack propagation
Numerical examples
Extension to 3D plane crack propagation (2)

- geometry and mesh

- notch tip curvature

non local + cohesive elements
→ about 530 000 dofs

crack plane
(cohesive interface)
Numerical examples
Extension to 3D plane crack propagation (3)

animation #3

[Valdenaire, 2011]
(EDF R&D internship)

\[ E = 30 \text{ GPa} \]
\[ \sigma_y = 3 \text{ MPa} \]
\[ \nu = 0.4 \]
\[ G_f = 0.1 \text{ N/mm} \]
\[ 2D = 100 \text{ mm} \]
Introduction
- Industrial context / scientific context
- Continuous and discontinuous approaches for fracture

1D analytical study: transition from non local damage growth to cohesive crack opening
- Continuous reference solutions for two regularization techniques
- Identification of an equivalent cohesive law
- Switching from non local damage to cohesive zone model

Extension to a 2D/3D FE framework
- FE implementation: explicit coupling algorithm
- 2D simulations
- 3D simulations

Conclusion and prospects
Conclusion and prospects
The proposed approach shows promising results:
- very slight dependency on the critical damage parameter
- remains consistent with both non local and cohesive stand-alone models
- no additional CPU cost
- is able to deal with snap-back and multi cracking
However, several limitations must be studied to open new prospects:

- extension to mixed mode loading
- crack-path tracking based on the spatial distribution of damage
- gradual insertion of the cohesive discontinuity (remeshing tool, XFEM, …)
- mesh adaptivity (reduction of the computational cost)
Thank you for your attention
[Cazes et al. 2008]

[Comi et al. 2007]

[Forest, 2009]

[Jirásek & Zimmermann 2001]

[Lorentz & Godard 2011]

[Mediavilla et al. 2006]

[Simone et al. 2003]

[Triantafyllidis & Aifantis 1986]