Multi-scale approach of the mechanical behavior of RC structures – Application to nuclear plant containment buildings

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Industrial context

- EDF is responsible for numerous reinforced concrete structures (dams, cooling towers, containment buildings…)

- Ensuring the safety of power plants
  - Evolution of the safety criteria
  - Extension of the operating life

- Anticipating the maintenance works
  - Minimizing hazards
  - Reducing maintenance costs

- Key role of modelling tools
Industrial context

- **Nuclear plant containment buildings**

- **Simple or double walled prestressed reinforced concrete shell**

- **Dual role**
  - Protecting the reactor core from external aggression
  - Preventing the escape of radioactive materials into the environment
Industrial context

- Focus on French nuclear power plants of type P4, P’4 and N4 (1350–1400 MWe)

- Double concrete shell without metal liner

- Safety criterion: at 5.3 bars, the leakage must be less than 1.125% of the contained gas mass during 24h

- Leaktightness measured experimentally every 10 years

- Major issue for the extension of the operating life
Industrial context

- Shrinkage and creep
- Reinforcements and tendons
- Steel-concrete bond

- Porosity of concrete
- Opening of cracks

Measured leakage
Industrial context

- Shrinkage and creep
- Reinforcements and tendons
- Steel-concrete bond

Porosity of concrete → Opening of cracks → Measured leakage
Research context

Different ways of modelling reinforcements and tendons

- 3D modelling
  - High computational cost
  - Not suitable for industrial applications

- Bar and beam elements
  - Spurious 1D-3D interaction  [Lorentz 05]
  - High stress concentrations

- Smeared reinforcements  [Cervenka 09]
  - Blurred representation
  - Need for a mathematical basis
Outline of the talk

1. Asymptotic models representing the reinforcements and tendons
2. Implementation and validation
3. Simulation of an experimental test
Asymptotic analysis

The reference model

- Stiff fibers periodically distributed on a surface in a matrix
  - Reinforcing bars
  - Prestressing tendons

- Elastic behaviour of the fibers and the matrix

- Possible decohesions between the fibers and the matrix
Asymptotic analysis

The asymptotic approach

- Study of the limit behavior of models

- Classical examples
  - Homogenization of microstructures
  - Boundary layers for fluids
  - Convergence of the numerical schemes

- Study of inhomogeneities located on a surface
  - Study of thin homogeneous layers  [Sanchez-Palencia 70, Caillerie 90, Licht 97…]
  - Study of heterogeneities periodically distributed  [Nguetseng 85, Abdelmoula 99]
Asymptotic analysis

Identification of the parameters

- Main parameters of the problem
  
  \[ d, \ s, \ L, \ E_F, \ E_M \]

- Dimensionless ratios

\[
\frac{d}{s} \sim \frac{1}{10} \\
\frac{s}{L} \sim \frac{1}{6} \\
\frac{E_M}{E_F} \sim \frac{1}{7}
\]
Asymptotic analysis (I)

Limit of small fibers

- Adherent interface between the fibers and the matrix

\[ \eta = \frac{s}{L} \rightarrow 0 \]

\[ \frac{E_M}{E_F} \text{ and } d \frac{d}{s} \text{ constant} \]
Asymptotic analysis (I)

Separation of scales

\[ \mathbf{u} = \mathbf{u}_0 + \eta \mathbf{u}_1 + \eta^2 \mathbf{u}_2 + \cdots \]

\[ \sigma = \sigma_0 + \eta \sigma_1 + \eta^2 \sigma_2 + \cdots \]

\[ \mathbf{u} = \mathbf{v}_0 + \eta \mathbf{v}_1 + \eta^2 \mathbf{v}_2 + \cdots \]

\[ \sigma = \tau_0 + \eta \tau_1 + \eta^2 \tau_2 + \cdots \]
Asymptotic analysis (I)

Influence of the fibers at order 0

Transmission conditions

\[
\begin{align*}
\begin{bmatrix} \mathbf{u}^0 \end{bmatrix} &= 0 \\
\begin{bmatrix} \sigma^0 \end{bmatrix} \cdot \mathbf{n} &= 0
\end{align*}
\]

- No jump of displacement and stress across the interface
- Necessary to go to next order

« Taylor’s theorem »
Asymptotic analysis (I)

Influence of the fibers at order 1

Transmission conditions

\[
\begin{align*}
\begin{bmatrix} u^1 \end{bmatrix} &= d \\
\begin{bmatrix} \sigma^1 \end{bmatrix} \cdot n &= -\text{div}_\Gamma(\sigma^\Gamma)
\end{align*}
\]

Effective behavior

\[
\begin{align*}
d &= L \cdot (\sigma^0 \cdot n) + M : \varepsilon_\Gamma(u^0) \\
\sigma^\Gamma &= - (\sigma^0 \cdot n) \cdot M + N : \varepsilon_\Gamma(u^0)
\end{align*}
\]

\(L, M\) and \(N\) characterize the effective behavior of the interface
Asymptotic analysis (I)

Influence of the fibers at order 1

Microscopic problems

\[
\begin{align*}
\tau^0 &= A : \left( \varepsilon_i (u^0) + \varepsilon_y (v^1) \right) \\
\text{div}_y (\tau^0) &= 0 \\
\lim_{\pm \infty} \tau^0 \cdot n &= \sigma^0 \cdot n \\
v^1 \text{ and } \tau^0 \text{ periodic}
\end{align*}
\]

\textbf{L, M and N} are identified once for all by solving 6 elementary problems
Asymptotic analysis (I)

Sequential resolution

- Solve the order 0 problem
- Not suitable for practical computations

- Extract \( \begin{cases} \sigma^0 \cdot n \\ \varepsilon_{\Gamma}(u^0) \end{cases} \)
- Two successive resolutions of the mechanical problem

- Obtain \( \begin{cases} [u^1] \\ [\sigma^1] \cdot n \end{cases} \)
- Infinite energy singularities in the order 1 problem

- Solve the order 1 problem
Asymptotic analysis (I)

Coupled formulation

- Define a problem whose solution is accurate *up to order 1*

- Interface behaviour coupling:
  - A membrane behaviour
  - An elastic interface behaviour

- Interface energy:

\[
\mathcal{E}_\Gamma(U, \Sigma) = \frac{1}{2} (\Sigma \cdot n) \cdot L \cdot (\Sigma \cdot n) + \frac{1}{2} \varepsilon_\Gamma(\bar{U}) : N : \varepsilon_\Gamma(\bar{U})
\]

\[
= \frac{1}{2} (\|U\| - M : \varepsilon_\Gamma(\bar{U})) \cdot L^{-1} \cdot (\|U\| - M : \varepsilon_\Gamma(\bar{U})) + \frac{1}{2} \varepsilon_\Gamma(\bar{U}) : N : \varepsilon_\Gamma(\bar{U})
\]
Asymptotic analysis (I)

Boundaries on the effective behavior

- Upper and lower bounds

\[ \varepsilon(u) : \langle A^m - A \rangle_Y : \varepsilon(u) \leq (\sigma \cdot n) \cdot L \cdot (\sigma \cdot n) \leq \sigma : \langle S - S^m \rangle_Y : \sigma \]

\[ \sigma : \langle S^m - S \rangle_Y : \sigma \leq \varepsilon_T(u) : N : \varepsilon_T(u) \leq \varepsilon(u) : \langle A - A^m \rangle_Y : \varepsilon(u) \]

- Stability of the coupled formulation

- Stiff inclusions

\[ E > E^m \implies \begin{cases} L < 0 \\ N > 0 \end{cases} \]

- Soft inclusions

\[ E < E^m \implies \begin{cases} L > 0 \\ N < 0 \end{cases} \]
Asymptotic analysis (I)

Application to steel reinforcements

- In practice, $L$ and $M$ are negligible
- No jump of displacement
- Simple orthotropic membrane model

\[
\varepsilon_\Gamma(U) = \frac{1}{2} \varepsilon_\Gamma(U) : N : \varepsilon_\Gamma(U)
\]

- The effective behavior is stable
Asymptotic analysis (I)

Analysis of the general asymptotic model

Pros

- Able to describe the effective behavior of a wide range of heterogeneities
- Accurate until order 1
- Gives access to the stress field near the heterogeneities

Cons

- Valid only for elastic behaviors
- Generally unstable
Asymptotic analysis (II)

Limit of small stiff fibers

\[ \frac{s}{L} \propto \frac{E_M}{E_F} \to 0 \]

\[ \frac{d}{s} \text{ constant} \]

Adherence law between the fibers and the matrix \( \tau(\delta) \)
Asymptotic analysis (II)

Kinematics of the fiber

- The possible decohesion introduces new kinematics of the fibers
  - Sliding of the fiber with respect to the matrix
  - Rotation of the fiber around its axis

- We decide to neglect the rotation of the fibers
Asymptotic analysis (II)

Grid model with decohesions

- Orthotropic membrane with unidirectional rigidity

- Interface law between the membrane and the matrix

\[ \kappa_m = \frac{\pi d^2}{4s} E_s \]

\[ \tau_m(\delta) = \frac{2\pi d}{s} \tau(\delta) \]
Asymptotic analysis (II)

Analysis of the grid model with decohesion

Pros

- Parameters of the model identified analytically
- Possible decohesion of the fibers
- Induces less stress concentrations than the bar and beam elements
- Based on a rigorous variational formulation
- Possible yielding of the fibers

Cons

- Smeared representation of the fibers
- No confining effect on the steel-concrete interface law
Implementation and validation

Porosity of concrete → Measured leakage → Opening of cracks

Shrinkage and creep

Reinforcements and tendons

Steel-concrete bond
Implementation and validation

Steel-concrete interface law

- Different models developed by Eligehausen, Cox, Dominguez and Richard

- Need for a simpler model

\[ \tau(\delta) = \tau_0 \frac{(\delta / \delta_0)^\alpha}{(1 + \delta / \delta_0)^\beta} \]

- 4 parameters: \( \delta_0, \tau_0, \alpha \) and \( \beta \)

- No unloading friction
Implementation and validation

Validation of the steel-concrete interface law

- Experimental pull-out tests performed by Eligehausen in 1983
Implementation and validation

Implementation of the asymptotic models

- General asymptotic model
- Orthotropic membrane elements
Implementation and validation

Implementation of the asymptotic models

- Stiff model with decohesions
- Modular implementation
  - Orthotropic membrane elements
  - Interface elements based on a mixed formulation (augmented lagrangian)
Implementation and validation

Validation on a flexural test

- Bending of a concrete plate with two layers of reinforcements
- Perfect adhesion of the bars

- Comparison between:
  - A 3D reference model
  - A pure concrete model
  - The grid model
  - The orthotropic membrane model
Implementation and validation

Validation on a flexural test

- Excellent results with the membrane model
- Satisfactory results with the grid model

<table>
<thead>
<tr>
<th>Model</th>
<th>Displacement</th>
<th>Discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference model</td>
<td>87.1 µm</td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td>119 µm</td>
<td>37 %</td>
</tr>
<tr>
<td>Grid model</td>
<td>84 µm</td>
<td>3.6 %</td>
</tr>
<tr>
<td>Membrane model</td>
<td>87.3 µm</td>
<td>0.2 %</td>
</tr>
</tbody>
</table>

(900 000 dof)
Implementation and validation

Validation on a pull-out test

- Pull-out of a set of reinforcing bars in a plate
- Non-linear adherence law between the bars and the concrete

Comparison between:
- A 3D reference model
- The grid model with decohesion
Implementation and validation

![Graphs showing sliding and load-displacement relationships with models for reference and grid with decohesion.](image)

(350 000 dof)

(75 000 dof)
Implementation and validation

Comparison between both asymptotic models

- Models implemented in the finite element code Code_Aster

- Orthotropic membrane model
  - Excellent results in the elastic regime
  - Restricted to elastic behaviors

- Grid model with decohesion
  - Validated in the elastic and decohesion regimes
  - More appropriate for civil engineering simulations
Simulation of an experimental test

The experiment PACE 1450

Mechanical behaviour of a portion of containment building in real scale
Simulation of an experimental test

Experiment carried out by the Karlsruhe Institut für Technologie (KIT), Germany
Simulation of an experimental test

Experimental tests performed

<table>
<thead>
<tr>
<th>Test</th>
<th>Pressure (bars)</th>
<th>Prestress level</th>
<th>Appearance of cracks</th>
<th>Measure of leakage</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN 1</td>
<td>5.3</td>
<td>100 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUN 2</td>
<td>5.3</td>
<td>80 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUN 3</td>
<td>5.3</td>
<td>60 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RUN 4</td>
<td>6</td>
<td>60 %</td>
<td>Appearance of cracks</td>
<td></td>
</tr>
<tr>
<td>RUN 5</td>
<td>6</td>
<td>60 %</td>
<td>Measure of leakage</td>
<td></td>
</tr>
<tr>
<td>RUN 6</td>
<td>6</td>
<td>60 %</td>
<td>Measure of leakage</td>
<td></td>
</tr>
</tbody>
</table>
Simulation of an experimental test

Modelling of the experiment

- Grid model for the reinforcements and horizontal prestressing tendons
- 3D modelling for the vertical tendon
- Elastic behavior for the concrete, with autogeneous and drying shrinkage
- Damage modeled through 5 cohesive cracks
Simulation of an experimental test

Discretization of the problem

- 250,000 dof
- Newton with line-search
- 2 minutes / Newton iteration
- 5 à 15 iterations / time step
- 50 time steps
- About 24 hours
Simulation of an experimental test

Modelling of RUN 4

- Appearance of two cracks for an internal pressure of 6 bars

- Appearance of four cracks for an internal pressure of 6 bars
Simulation of an experimental test

Modelling of RUN 6

- Good estimation of the crack openings
- Significant influence of shrinkage
- Dissymmetry in the measurements
  - steel-concrete interface friction
  - roughness of the crack lips
Simulation of an experimental test

Modelling of the leaktightness

Hagen-Poiseuille laminar flow

<table>
<thead>
<tr>
<th>Pressure (bars)</th>
<th>1</th>
<th>4</th>
<th>4.9</th>
<th>5.3</th>
<th>6</th>
<th>5.3</th>
<th>4.9</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured leak rate (g/s)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>33</td>
<td>22</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Estimated leak rate (g/s)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>9.8</td>
<td>1300</td>
<td>9.9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Conclusion and perspectives

From asymptotic analysis to industrial applications

- Development of innovative asymptotic models to represent reinforcements and tendons
- Implementation and validation of these models with gradual integration of non-linearities
- Proposal of a modelling strategy to predict the leaktightness of containment buildings
Conclusion and perspectives

Future Works

- Theoretical developments
  - Instability of the general asymptotic model
  - Modelling of the confinement effect

- Numerical simulations
  - Improvement of the line-search algorithm
  - Use of concrete damage models

- Estimation of the leak rate
  - Improvement of the flow rule in the cracks
    - [Rizkalla 84, Suzuki 92]
  - Topology of the crack network in the structure
Thank you very much for your attention
Additional slides

Instability of the general asymptotic model

(a)

(b)

(c)
Additional slides

Instability of the general asymptotic model

Instability of the elastic interface behavior

Instability of the membrane behavior
Additional slides

Application to steel reinforcements

\[
L = \frac{S_i}{E_M} \mathbf{e}_i \otimes \mathbf{e}_i
\]

\[
M = c_\alpha \mathbf{e}_1 \otimes (\mathbf{e}_\alpha \otimes \mathbf{e}_\alpha)
\]

\[
N = E_M R_{\alpha\beta} (\mathbf{e}_\alpha \otimes \mathbf{e}_\alpha) \otimes (\mathbf{e}_\beta \otimes \mathbf{e}_\beta)
\]

\[
+ 4 E_M G (\mathbf{e}_\alpha \otimes_s \mathbf{e}_\beta) \otimes (\mathbf{e}_\alpha \otimes_s \mathbf{e}_\beta)
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(s_1)</th>
<th>(s_\alpha)</th>
<th>(c_\alpha)</th>
<th>(R_{\alpha\alpha})</th>
<th>(R_{23})</th>
<th>(G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>-0.017</td>
<td>-0.050</td>
<td>-0.002</td>
<td>0.056</td>
<td>0.006</td>
<td>0.009</td>
</tr>
</tbody>
</table>
Additional slides

Steel-concrete interface law

Fissures coniques

Béton écrasé

Béton fragmenté

[Dominguez 05]
Additional slides

Intersection between a crack and the grid model
Additional slides

Modelling of an open crack

- (260 000 dof)
- (140 000 dof)
Additional slides

Before crack opening

After crack opening

Martin David - Multi-scale approach of the behavior of RC structures - 19/06/2012
Additional slides

Talon-Curnier interface law