A new marching ridges algorithm for crack path tracking in regularized ductile media

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Introduction

1. State of the art
   a. Discontinuity modeling
   b. Crack path tracking
   c. Motivation

2. Principle
   a. Ridge definition
   b. Marching ridges process
   c. In 3D

3. Numerical results
   a. Strategy
   b. Small strain framework
   c. Finite strain framework

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Introduction

Context:
Predictive numerical simulation of **ductile failure** for the design of industrial structures in the aerospace domain

[Ductile failure of a specimen strained axially](http://spaceflight.esa.int)
Schematically at the microscale

A. Weck, D. Wilkinson,
McMaster University, Canada
Schematically at the microscale

A. Weck, D. Wilkinson, McMaster University, Canada
Void coalescence and new free surface creation

Schematically at the microscale

A. Weck, D. Wilkinson, McMaster University, Canada
Introduction

Context :
Predictive numerical simulation of **ductile failure** for the design of industrial structures in the aerospace domain

Ductile failure of a specimen strained axially

[http://spaceflight.esa.int]
Introduction

Objective:
Modeling material degradation and crack propagation with a continuous discontinuous approach

1. Continuous description up to the onset of fracture
Inobjectivity with standard strain-softening continuum must be taken into account!
Objective:
Modeling material degradation and crack propagation with a continuous discontinuous approach

1. Continuous description up to the onset of fracture

2. Discontinuous description for crack propagation
1. How?  →  Discontinuity modeling

2. Where?  →  A crack path tracking algorithm  
               (E-FEM, X-FEM, mesh adaption)

3. When?  →  Difficult matter...  →  - energy conservation
             - consistency with experiments...
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Conclusion and outlook
1. State of the art

a. Discontinuity modeling (for FEM)

Degradation modelization encompassed?

- yes TLS, CZM
- no

Additional element enrichment?

- yes E-FEM, X-FEM
- no

Loss of mass?

- yes Kill element
- no Mesh adaption
1. State of the art

a. Discontinuity modeling (for FEM)

Degradation modelization encompassed?

- yes
- TLS, CZM
- no
1. State of the art

a. Discontinuity modeling (for FEM)

Discontinuity + degradation modelization:

The *Thick Level Set damage model* [Moës et al 11, Bernard et al 12] quasi-brittle

Plus: supra-elemental approach (contains a non local treatment)

Limitation: iso-values of a level set with $\text{grad } d = \text{cste} + \text{surface/volume } d=1$

Fig. 1. A single level set function locating two damaged zones. The iso-zero $\Gamma_0$ locates the transition between the same and damaged material. As the damage growth, the iso-1 appears, locating transition between damaged and fully damaged zones. This zone where $d = 1$ can be seen as a macro crack.
1. State of the art

a. Discontinuity modeling (for FEM)

Discontinuity + degradation modelization:

The *Thick Level Set damage model* [Moës et al 11, Bernard et al 12] quasi-brittle
Plus: supra-elemental approach (contains a non local treatment)
Limitation: iso-values of a level set with $\text{grad } d = \text{cste } + \text{surface/volume } d=1$

*Cohesive zone models*

inter-elemental approaches

2D: [Needleman 87, Camacho and Ortiz 96]
3D: [Ortiz and Pandolfi 99, Pandolfi and Ortiz 02]

Possible equivalence with the non local damage law [Cazes et al 08, Cuvilliez 12]
Limitation: Mesh dependence if not coupled with adaptive remeshing techniques
1. State of the art

a. Discontinuity modeling (for FEM)

Degradation modelization encompassed?

- yes
  - TLS, CZM
- no
  - Additional element enrichment?
    - yes
      - E-FEM, X-FEM
    - no
1. State of the art

a. Discontinuity modeling (for FEM)

Discontinuity - independently of the constitutive behavior

Intra-elemental methods with displacement modes enrichment

- *Embedded (strong) discontinuity Finite Element Methods (E-FEM)*
  
enrichment of elements for displacement jump [Jirasek and Zimmermann 01, Simo et al 93, Oliver 96, Armero and Garikipati 96]

Limitations: limited kinematic properties
1. State of the art

a. Discontinuity modeling (for FEM)

Discontinuity - independently of the constitutive behavior

- Intra-elemental methods with displacement modes enrichment

  - *Embedded (strong) discontinuity Finite Element Methods (E-FEM)* 
    enrichment of elements for displacement jump

  - *eXtended Finite Element Method (X-FEM)* [Belytschko and Black 99, Moës et al, 99] 
    enrichment of nodes

2D quasi-brittle [Patzak and Jirasek 03, Simone et al 03, Comi et al 07] 
2D viscoplastic [Wells and Sluys 02]; ductile failure: [Simatos 10] 
3D [Sukumar et al 00, Moës et al 02, Gravouil et al 02, Gasser and Holzapfel 05-06, Areias and Belytschko 05, Simatos 10]
1. State of the art

a. Discontinuity modeling (for FEM)

Degradation modelization encompassed?

- yes TLS, CZM
- no

Additional element enrichment?

- yes E-FEM, X-FEM
- no

Loss of mass?

- yes
- no

Kill element

- yes

Mesh adaption

- no
1. State of the art

a. Discontinuity modeling (for FEM)

Discontinuity - independently of the constitutive behavior
- independently of the finite element formulation

- *Kill element technique* [Mahishi and Adams 82, Bouchard 05]
  rather simple to implement
  mesh dependence
  non conservation of material or + edge smoothing [Labergere 14]

- *Mesh adaption*
  linear fracture mechanics: [Carter et al 00, Chiaruttini et al 10]
  ductile: 2D [Mediavilla 05], 3D [Peerlings et al 08, Javani 11]
  important within the finite strain framework: correct shape factor
  + error estimator for mesh optimality
  field transfer
1. State of the art

a. Discontinuity modeling (for FEM)

Degradation modelization encompassed?

- yes TLS, CZM
- no

Additional element enrichment?

- yes E-FEM, X-FEM
- no

Loss of mass?

- yes
- no Mesh adaption
- no kill element
1. State of the art

Transition from damage to fracture:

1. How? → Discontinuity modeling

2. Where? → A crack path tracking algorithm
   (E-FEM, X-FEM, mesh adaption)

3. When? Difficult matter... → - energy conservation
   - consistency with experiments...
1. State of the art

b. Crack path tracking

3 groups [Oliver et al 04]:

- **Local (or propagation) algorithms**
  geometrical propagation of the crack from an initial front
  local informations – cost effective

- **Global tracking algorithms**
  trace all possible crack paths at once
  need to solve an additional global – cumbersome and code-invasive
  ex: heat conduction-like problem [Oliver et al 04]
  small def [Huespe et al 09], finite strain [Huespe et al 12]

- **Tracking algorithms based on the level set concept**
  [Stolarska et al 01, Moës et al 11]
  share some of the characteristics of local and global algo.
1. State of the art

b. Crack path tracking

*Local (or propagation) algorithms*

gеometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the formulation type:

a. Local formulations: loss of ellipticity of the equations
   i) bifurcation analysis + ii) orientation criterion

b. Regularized formulations: uniqueness of the solution
   i) orientation criterion + ii) insertion criterion
1. State of the art

b. Crack path tracking

Local (or propagation) algorithms

geometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the formulation type:

a. Local formulations: loss of ellipticity of the equations
   i) bifurcation analysis + ii) orientation criterion

-> [Areias and Belytschko 05]:
   “viscosity-regularized” damage model (quasi-brittle case)
   i) loss of stability study
   ii) spectral decomposition of an averaged strain
       (+ cohesive law + X-FEM)
1. State of the art

b. Crack path tracking

*Local (or propagation) algorithms*

geometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the formulation type:

a. Local formulations: loss of ellipticity of the equations
   i) bifurcation analysis + ii) orientation criterion

-> [Oliver et al 14]:
   i) 2 admissible local directions
-> [Oliver + Huespe 04, Oliver et al 10]
   ii) selection of the best direction: crack-path-field technique

   -> resolution of a secondary problem which involves smoothing of both the
      localized scalar field and its directional derivative

(+enrichment of the elements)
1. State of the art

b. Crack path tracking

*Local (or propagation) algorithms*

geometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the formulation type:

a. Local formulations: loss of ellipticity of the equations
   i) bifurcation analysis + ii) orientation criterion

-> **Limitation: cost**
1. State of the art

b. Crack path tracking

*Local (or propagation) algorithms*

geometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the representation of the localization band:

b. Regularized formulations: uniqueness of the solution

- non-local models [Eringen 66, Bazant and Jirasek 02, Peerlings 99,…]
- gradient-regularized models [Aero and Kuvshinkskii 60, Gao et al 99,…]
- Cosserat models [Mühlhaus and Vardoulakis 87, de Borst 91, …]
1. State of the art

b. Crack path tracking

*Local (or propagation) algorithms*

gеometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the representation of the localization band:

b. Regularized formulations: uniqueness of the solution

i) orientation criterion

-> Weighted average - **Limitation: propagation in a single direction!**

Maximum effective stress [Wells and Sluys 02]

Maximum accumulation of the nonlocal equivalent strain [Simone et al 03]

Maximum damage [Mediavilla 05, Javani 11]

+ ii) insertion criterion
1. State of the art

b. Crack path tracking

*Local (or propagation) algorithms*

geometrical propagation of the crack from an initial front
local informations – cost effective

The choice depends on the representation of the localization band:

b. Regularized formulations: uniqueness of the solution
   i) orientation criterion

-> Weighted average - Limitation: propagation in a single direction

-> [Feld-Payet 10]: a marching ridges algorithm
to simultaneously detect several local maxima

+ ii) insertion criterion
1. State of the art

c. Motivation
A marching ridges algorithm for regularized formulations [Feld-Payet 10]

-> A new crack path tracking algorithm able to simultaneously detect several local maxima of a degradation-related variable by following the ridge line

Ridge detection (computer vision)

Marching ridges algorithms
[Thirion and Gourdon 93]
[Furst and Pizer 01]
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Conclusion and outlook
2. Principle

a. Ridge definition

Generalized height ridge definition [Eberly et al. 94, Furst and Pizer 98]

An \((d - 1)\)-dimensional height ridge of a function \(f : \mathcal{X} \rightarrow \mathcal{R}\), is a \((d - 1)\) locus corresponding to a relative maximum of \(f\) in a direction \(\mathbf{v}\) transverse to the putative ridge at a location \(\mathbf{x}\), that is:

\[
\begin{align*}
\mathbf{v} \cdot \nabla(f)(\mathbf{x}) &= 0 \\
\mathbf{v}^t \mathcal{H}(f)(\mathbf{x}) \mathbf{v} &< 0
\end{align*}
\]

where \(\mathcal{H}\) designates the Hessian matrix.
2. Principle

b. Marching ridges process

→ A local process:

1. Define a starting point
   Point of maximum damage

2. Define a test direction $\mathbf{V}$
   NEW!

3. Derivative evaluation
   LESS EXPENSIVE!

4. Expansion and marching
2. Principle

b. Marching ridges process

2. Define a test direction

\[ \mathbf{v} = \mathbf{e}_\theta \]
2. Principle

b. Marching ridges process

2. Define a test direction

NEW!

\[ \mathbf{v} = \mathbf{e}_\theta \]

3. Derivative evaluation

\[
\begin{align*}
\frac{\partial f}{\partial \theta} (\mathbf{x}_0 + R\mathbf{e}_r) &= 0 \\
\frac{\partial^2 f}{\partial \theta^2} (\mathbf{x}_0 + R\mathbf{e}_r) &< 0
\end{align*}
\]

\[ \rightarrow \text{LESS EXPENSIVE:} \]

\[
\begin{align*}
\mathbf{e}_\theta (\theta - \Delta \theta/2) \cdot \nabla f \left( \mathbf{x}_0 + R\mathbf{e}_r (\theta - \Delta \theta/2) \right) &> 0 \\
\mathbf{e}_\theta (\theta + \Delta \theta/2) \cdot \nabla f \left( \mathbf{x}_0 + R\mathbf{e}_r (\theta + \Delta \theta/2) \right) &< 0
\end{align*}
\]
2. Principle

b. Marching ridges process

\[
\frac{\partial f}{\partial \theta} = R \mathbf{e}_\theta \cdot \nabla f
\]
changes its sign from positive to negative.
2. Principle

b. Marching ridges process

Main advantages of this gradient-type criterion:

- extrema: min or max
- study of a scalar field
- simultaneous detection of several maximum damage directions necessary when a crack initiates completely inside the structure
2. Principle

c. In 3D

1. Define a starting point
   Point of maximum damage

   **Define a starting line**
   NEW!

2. Define a test direction
   idem

3. Derivative evaluation
   idem

4. Expansion and marching
2. Principle

c. In 3D

1. Define a starting point
   Point of maximum damage

Define a starting line

NEW!

Use the marching ridges algo in 3 orthogonal planes

Choose one segment among the possible directions
2. Principle

c. In 3D

Intersection with \( (I_{xy}) \)
Intersection with \( (I_{xz}) \)
Intersection with \( (I_{yz}) \)
2. Principle

c. In 3D

1. Define a starting point
   Point of maximum damage

   Define a starting line
   NEW!

2. Define a test direction
   idem

3. Derivative evaluation
   idem

4. Expansion and marching

Search plane \( i \)  
Back to 2D search!
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Conclusion and outlook
3. Numerical results

a. Strategy
3. Numerical results

a. Strategy

Regularization: non local implicit gradient models [Peerling PhD thesis 99]

Any type of regularised model with a variable describing degradation can be used
3. Numerical results

a. Strategy
3. Numerical results

a. Strategy
Discretization is fine enough to provide accurate damage description
→ mesh adaptivity based on a damage-driven incremental ZZ2-type error estimator
3. Numerical results

a. Strategy

-> Object of the second part
3. Numerical results

a. Strategy

- Element Formulation + Constitutive Behavior
- Error Estimation + Remeshing Criterion
- Ridge Tracking Algorithm
- Adaptive Remeshing + Field Transfer

Mesh + Internal Variable History

Size Map

$\text{crack increment}$
3. Numerical results

a. Strategy

Adaptive remeshing for crack insertion

Field transfer

- Nodes: nodal interpolation
- Integration points: moving least square

-> [Chiaruttini et al 10]
3. Numerical results

b. Small strain framework

**Model:**
small strain implicit gradient-enhanced strain-softening continuum model, featuring visco-elastoplasticity coupled with damage

[Engelen, Geers, Baaijens, Int. J. Plasticity, 2003]

**Finite elements:**
4 field mixed nonlocal elements for small strain to prevent mesh dependence and volumetric locking

[Feld-Payet, Besson, Feyel, Int. J. Damage Mechanics, 2011]
3. Numerical results

b. Small strain framework
3. Numerical results

b. Small strain framework

Improvement:

local remeshing only where \( \frac{h_{new}}{h_{old}} \leq X \)

or where a crack increment should be inserted

\( wp \)

0.75

0
3. Numerical results

b. Small strain framework

**Improvement:**

local remeshing only where \( \frac{h_{new}}{h_{old}} \leq X \)

or where a crack increment should be inserted

+ limit transfer induced errors \( \rightarrow \) mesh adaption can be considered more frequently
3. Numerical results

b. Small strain framework
3. Numerical results

c. Finite strain framework

Model:
GTN framework: porosity
Regularisation with 2 non local lengths for internal necking and localization

[Besson, CRAC 2013]

Finite elements:
3 field nonlocal elements for finite strain to prevent mesh dependence

[Besson, CRAC 2013]
c. Finite strain framework

Remeshing is necessary to preserve a correct shape factor for the elements and thus a good accuracy.

\[ \sigma_{22} \]

Reference

After 17 remeshing steps at given \( \Delta t \)

Without remeshing

Remeshing on initial configuration

Remeshing on current configuration
3. Numerical results

c. Finite strain framework

Remeshing is necessary to preserve a correct shape factor for the elements and thus a good accuracy.
3. Numerical results

c. Finite strain framework
3. Numerical results

c. Finite strain framework

Experimentally observed cup-cone and slant fractures: (c,d) Anisotropic material; the arrow indicates a secondary deformation band.

[Besson et al, Int. J. Solids and Structures, 2001]
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Conclusion and outlook
Conclusion

In this strategy to model ductile failure using mesh adaptivity:
- crack path tracking algorithm applicable to any regularised material behavior with a scalar material degradation variable

Numerical results for mode I-II ductile fracture within:
- the small strain framework in 2D and 3D
- the finite strain framework in 2D
Outlook

- Completing post-processing version
- Fully 3D simulations within the finite strain framework
- More complex crack path: mode III failure
- Crack insertion criterion / cohesive zone models
- Parameters identification with experimental comparison

(thèse Onera F. Bettonte 2014-2017)
A new marching ridges algorithm for crack path tracking in regularized ductile media

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Thank you for your attention

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